Inverse Scattering Transform (IST) for the Multicomponent Nonlinear Schrodinger (NLS) Equation Under Non-Vanishing Boundary Conditions

M.J. Ablowitz, G. Biondini, B. Prinari* and A.D. Trubatch

* Department of Mathematics, University of Colorado at Colorado Springs
1420 Austin Bluffs Pkwy, Colorado Springs - CO 80918 U.S.A.
Tel: 719-255-3522, email: bprinari@uccs.edu

Abstract:

This talk reports on the development of the IST for vector NLS under nonvanishing boundary conditions (NBCs) for an arbitrary number N of components. For the scalar NLS equation, which is termed here the $N = 1$ case, the scattering problem with NBCs is already complicated by the fact that the scattering parameter k "live" on a two-sheeted Riemann surface; however one still has two complete sets of analytic scattering functions. When $N > 1$, an additional complication arises: $2(N-1)$ out of the $2(N+1)$ scattering eigenfunctions are not analytic on either sheet of the Riemann surface, and one has to suitably complete the basis. The $N = 2$, or the 2-component case (so-called Manakov system) is somehow special. The IST for the Manakov system under NBCs was developed in 2006 [1] and the basic idea was to use the adjoint scattering problem to construct two additional analytic eigenfunctions. This technique, however, does not admit an obvious generalization to an arbitrary number of components. Moreover, when $N > 3$ yet another complication is added: the eigenvalue associated to the nonanalytic scattering eigenfunctions becomes a multiple eigenvalue, with multiplicity $N - 2 > 1$. In order to obtain complete the basis of analytic eigenfunctions for the general multicomponent scattering problem, we generalize the approach suggested in [2] for general scattering and inverse scattering on the line but developed under the assumption of vanishing BCs. The key step is the introduction of a fundamental tensor family as solutions of a suitable scattering problem associated to the given one, in such a way that each tensor is analytic on either one or the other sheet of the Riemann surface. Then we show that it is possible to algorithmically reconstruct the fundamental matrices of solutions of the scattering problem from the fundamental tensors, and to establish their analyticity properties.

References:

1. B. Prinari, M.J. Ablowitz and G. Biondini, J. Math. Phys. 47, 063508 (2006).
2. R. Beals, P. Deift and C. Tomei, Direct and inverse scattering on the line, Mathematical Surveys and Monographs, No. 28. AMS, Providence, (1988).